Proceedings of the Eighteenth Ramanujan Symposium On

Recent Trends in Dynamical Systems and Mathematical Modelling,

University of Madras, Chennai,

25–27, September, 2013 Volume 18 (2014), 81–86

Analysis of Two- Echelon Inventory System with Retrial Demand

M. Rameshpandi, R.Satheeshkumar, and K.Krishnan

Abstract.

In this article, we consider a continuous review inventory system with Markovian demand. The operating policies are (s, S) and (0, M) policy, that is the maximum inventory level at lower echelon is S and whenever the inventory drops to s, an order for Q(= S-s,) units is placed at the same time in the higher echelon , the maximum inventory level is fixed as M(= nQ: n =1,2, ..) and has an instantaneous replenishment facility from an abundant supply source. The ordered items are received after a random time which is assumed to be exponential distribution. The demands that occur during stock out period are enter into the orbit of finite size. These orbiting demands retry for their demand after a random time, which is assumed to be exponential distribution. The joint probability distribution of the inventory level at lower echelon, higher echelon and the number of customer in the orbit is obtained in the steady state case. Various system performance measures in the steady state are derived and the long run total expected cost rate is calculated. Various system performance measures are derived and the results are illustrated numerically.

Keywords.

Continuous review inventory system, Two-echelon, (s, S) policy, retrial demand.

2010 MSC.

1. Introduction

Controlling inventories in supply chain is recognized as more important than purchasing raw materials. Inventory affects costs in more ways than anybody would realize. Understanding and managing inventory driven costs can have a significant impacts as margins. To make PC business more cost competitive HP's Strategic Planning and Modeling (SPaM) group lead by Corey Billingten undertook an exhaust review of the PC business overall cost structure in 1977. He identified the problem that mismatches between demand and supply leading to excess inventory. Cohen and Lee (1988) develop a model for establishing material requirements policy for all materials for every stage in the supply chain production system. In this work the authors use four different cost based sub models (i) material control (ii) production control (iii) Finished goods stockpile and (iv) Distribution. Each of these sub models is based on a minimum-cost objective. Svoronos and Zipkin (1991) considers multi-echelon distribution type supply chain systems. In this article the authors assume a base stocks, one-for-one (S-1,S) replenishment policy for each facility and that demand for each facility follow an independent Poisson process. They obtained steady state approximations for the average inventory level and the average number of outstanding back orders at each location for any choice of base stock level.

Pyke and Cohen (1993) [10] develop a mathematical programming model for an integrated supplychain, using stochastic sub-models to calculate the values of the random variables included in the mathematical program. The authors consider a three-level supply chain, consisting of one productone manufacturing facility, one warehousing facility and one retailer. The model minimizes total cost subject to a service level constraint and holds the set-up times, processing times, and replenishment lead times constant for a particular production network.

Pyke and Cochen (1994) [11] follow the Pyke and Cochen (1993) research by including a more complicated production network. In Pyke and Cochen (1994) the authors again consider an integratedsupply chain with one manufacturing facility, one warehouse and one retailer but now multiple product types. The new model yields similar outputs however it determines the key decision variables for eachproduct type. This model yields the approximate economic (minimum cost) reorder interval.

(for eachproduct type), replenishment batch sizes (for each product type) and the order-up-to product levels (for the retailer, for each product type) for a particular supply chain network. Finally Lee et al. (1977) [9] develop stochastic mathematical models describing ”The Bull whip Effect”, which is defined as aphenomenon in which the variance of buyer demand becomes increasingly amplified and distorted at echelon upwards throughout the supply chain. In this research the author develop stochastic analytic models describing the four causes of the Bull whip effect (demand signal processing, rationing game, order batching and price variations), and show how these causes contribute to the effect.

A complete review was provided by Benita M. Beamon (1998)[2]. Recently however, there has been increasing attention placed on performance, design and analysis of the supply chain as a whole. HP's (Hawlet Packard) Strategic Planning and Modeling (SPaM) group initiated this kind of research in 1977. From practical stand point the supply chain concept arose from a number of changes in the manufacturing environment, including the rising costs of manufacturing, the shrinking resources of manufacturing bases, shortened product life cycles, the leveling of planning field within manufacturing, inventory driven costs (IDC) involved in distribution (2005) [7] and the globalization of market economies. Within manufacturing research, the supply chain concept grow largely out of two-stage multi-echelon inventory models, and it is important to note that considerable research in this area is based on the classic work of Clark and Scarf (1960) [5].

A complete review on this development were recorded by Federgruen (1993) [6]. Recent developments in two-echelonn models may be found in Q.M. He, and E.M.Jewkes (2000) [12] and Buzzacott(1966) [3] and Axaster. S. (1993a) [1]. This paper deals with a simple supply chain that is modeled asa single warehouse and two-retailer system handling a single product. In order to avoid the complexity, we assumed the two-echelon inventory system. This restricts our study to design and analyze anetwork of inventory, which is a building block of the whole supply chain system. The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, both transient and steady state analysis are done. Section four deals with the operating characteristics of the system and section 5, the cost analysis for the operation. In section 6 some numerical examples are given and the last section 7, concludes the paper.

Notation:

• $[A]_{i,j}$: (i, j) th element/block of the matrix A

• I_n : Identity matrix of order n

• e : column vector of ones with appropriate dimension

• S = The maximum inventory level at retailer nodes

• s = Reorder level at retailer nodes

• $Q = S - s$

• $E = \{(i, j, k) | i = 0, 1, \ldots, N, j = 0, 1, \ldots, S, k = Q, 2Q, , \ldots, nQ\}$

• $\sum_{k=Q}^{nQ}(.)$ stands for $\sum_{k=Q}(.) + \sum_{k=2Q}(.) + \sum_{k=3Q}(.) + \ldots + \sum_{k=nQ}(.)$

2. Model Description

In this thesis, we consider a two level supply chain consisting one product, one manufacturing facility, one warehousing facility and one retailer. The demands initiated at retailer node follow Poisson process with parameter $\lambda(> 0)$ and the lead times are exponentially distributed with parameter $\mu(> 0)$. The retailer follows (s, S) policy for maintaining his inventory and the distributor follow (0, nQ) policy for maintaining his inventory. The unsatisfied customers are treated as retrial customers and they are waiting in the orbit with finite capacity N . The repeated customers from the orbit (with capacity i) are entered into the system with rate $i\theta(> 0)$. Even though we have adopted two different policies in the supply chain, the distributors policy is depends upon the retailers policy. The model minimizes the total cost incurred at all the locations subject to the service level constraints. The system performance measures and the total cost are computed in the steady state.

3. Analysis

Let $X(t)$, $Y(t)$ and $Z(t)$ respectively denote the number of demands in the orbit, the on hand inventory level in the retailer node and the number of items in the Distribution centre at time t. From the assumptions on the input and output processes, clearly $X(t) = \{(X(t), Y(t), Z(t)) : t > 0\}$ is a Markov process with state space E. the infinitesimal generator of this process. The infinitesimal generator of this process $A = (a(i, k, m : j, l, n)), (i, j, m), (j, l, n) \in E$ can be obtained from the following arguments.

- The primary arrival of demand to the retailer node makes a transition in the Markov process from (i, j, k) to $(i - 1, j, k)$ with intensity of transition λ.
- The arrival of a demand at retailer node from orbit transition in the Markov Process from (i, j, k) to $(i - 1, j - 1, k)$ with intensity of transition $i\theta$.
- Replenishment of inventory at retailer node makes a transition from (i, j, k) to $(i, j + Q, k - Q)$ with rate of transition μ

Then, the infinitesimal generator has the following finite QBD structure: