STOCHASTIC ANALYSIS OF PERISHABLE INVENTORY SYSTEM IN TWO-ECHELON

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Abstract

In this paper we consider a continuous review inventory control system to Multi-echelon system, which is a building block for Supply Chain. The integrated inventory control system consists of a Manufacturer (MF), single Warehouse (WH), one Distribution Centre (DC) and **n** identical retailers system handling a single perishable product. A (s, Q) type inventory system with Poisson demand and exponentially distributed lead times for items are assumed at DC (middle echelon). An one-for-one type inventory policy is assumed at retailers node (lower echelon). Demands occurring during the stock out periods are assumed to be lost. The WH (upper echelon) replenishes their stocks from Manufacturer, which has abundant stocks for supply. The items are supplied to the Manufacturer in packs of Q items from the warehouse. The perishable product has the exponential decay of constant rate. Here the exposure of items occurs only at retailer node. The steady state probability distribution and the operating characteristics are obtained explicitly. The required algorithm is designed and it is executed.

Keywords: Supply Chain, Markov process, Inventory control, Optimization.

1. Introduction

Supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products and the distribution of these finished products to customers. Supply Chain exists in both service and manufacturing organizations, but the complexity of the chain may vary greatly from industry to industry.

Inventory decision is an important component of the supply chain management, because Inventories exist at each and every stage of the supply chain as raw material or semi-finished or finished goods. They can also be as Work-in-process between the stages or stations. Since holding of inventories can cost anywhere between 20% to 40% of their value, their efficient management is critical in Supply Chain operations

The usual objective for a multi-echelon inventory model is to coordinate the inventories at the various echelons so as to minimize the total cost associated with the entire multi-echelon inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. The reason is that a key concept of supply chain management is that a company should strive to develop an informal partnership relation with its suppliers and retailers that enable them jointly to maximize their total profit.

Information technology has a substantial impact on supply chains. Scanners collect sales data at the point-of-sale, and Electronic Data Interchange (EDI) allows these data to be shared immediately with all stages of the supply chain.

Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature. As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency.

The first quantitative analysis in inventory studies started with the work of Harris (1915) [9].Clark and Scarf (1960) [4] had put forward the multi-echelon inventory first. They analyzed a N-echelon pipelining system without considering a lot size, Recent developments in two-echelon models may be found in Q.M. He and E.M. Jewkes (2000)[14].Sven Axsäter (1990)[1] proposed an approximate model of inventory structure in SC. One of the oldest papers in the field of continuous review multi-echelon inventory system is a basic and seminal paper written by Sherbrooke [15] in 1968. He assumed (S-1, S) polices in the **Deport-Base** systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

Continuous review models of multi-echelon inventory system in 1980's concentrated more on repairable items in a **Depot-Base** system than as consumable items (see Graves [6,7], Moinzadeh and Lee [12]). All these papers deal with repairable items with batch ordering. Jokar and Seifbarghy [14] analyzed a two echelon inventory system with one warehouse and multiple retailers controlled by continuous review (R, Q) policy. A Complete review was provided by Benita M. Beamon (1998)[2]. the supply chain concept grow largely out of two-stage multi-echelon inventory models, and it is important to note that considerable research in this area is based on the classic work of Clark and Scarf (1960)[4]. Continuous review perishable inventory models studied by Kalpakam. S and Arivarignan . G (1998)[10] and a continuous review perishable inventory system at Service Facilities was studied by Elango (2001) [5]. A continuous review (s, S) policy with positive lead times in two-echelon Supply Chain for both perishable and non perishable was considered by Krishnan. K and Elango. C. 2005 [11].

The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, both transient and steady state analysis are done. Section 4 deals with the derivation of operating characteristics of the system and section 5 deals with the algorithm for this model. In section 6, deals with the cost analysis for the operation. Numerical examples and sensitivity analysis are provided in section 7 and the last section 8 concludes the paper.

2. The Model description

The inventory control system in supply chain considered in this paper is defined as follows.

We consider a supply chain system consisting of a manufacturer, warehousing facility, single distribution centre and **n** identical retailers dealing with a single finished product. These finished products moves from the manufacturer through the network consist of WH, DC, Retailers then the final customer.

A finished product is supplied from MF to WH which adopts (0, M) replenishment policy then the product is supplied to DC who adopts (s, Q) policy. The demand at retailers node follows a independent Poisson distribution with rate λ_i (i = 1, 2, 3, … , n). Scanners collect sales data (demand or perish of an item) at retailer nodes and Electronic Data Interchange (EDI) allows these data to be shared to DC.

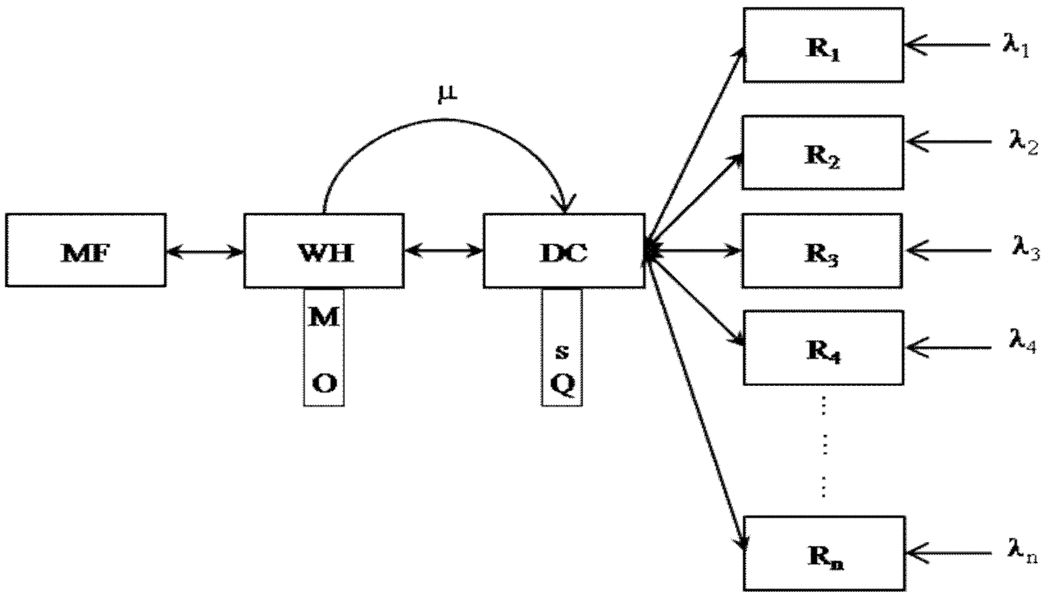

Figure 1 - Multi-echelon Inventory System

With the strong communication network and transport facility a unit of item is transferred to the corresponding retailer with negligible lead time. That is all the inventory transactions are managed by DC. Supply to the Manufacturer in packets of Q items is administrated with exponential lead time having parameter μ (>0). The replenishment of items in terms of pockets is made from Manufacturer to WH is instantaneous. Demands occurring during the stock out periods are assumed to be lost. The maximum inventory level at DC node S is fixed and the reorder point is s and the ordering quantity is Q(=S-s) items. The maximum inventory level at Manufacturer is M (M=nQ).

The life time of each item supplied is exponentially distributed with parameter γ (> 0). It is assumed that the item in stock perishes only at the time of handling them at a retailer node; Decay during lead time is neglected because of tamper free packages. Decay is not considered at warehouse and distribution centre inventory.

According to the assumptions the on hand inventory levels at both nodes follows a random process.

We fix the following notations for the forthcoming analysis part of our paper.

$[\boldsymbol{R}]_{ij}$: The element /sub matrix at $(i,j)^{th}$ position of $\boldsymbol{R}$.

0 : Zero matrix.

I : Identity matrix.

e : A column vector of 1's of appropriate dimension.

$\mathbf{I_i(t)}$: On hand inventory level at time t at location i(i =0,1) .

γ_i : The Perishable rate at retailer node i (i =0,1).

$\mathbf{k_D}$: Fixed ordering cost, regardless of order size at DC node.

$\mathbf{k_R}$: $\mathbf{k_R} = \bar{k}_i$, where i =1,2,3 … n. the average ordering cost at retailer nodes.

$\mathbf{k_0}$: $\mathbf{k_D + k_R}$, ordering cost for integrated DC system.

$\mathbf{k_1}$: Fixed ordering cost for WH.

$\mathbf{h_D}$: The holding cost per unit of item per unit time at DC

$\mathbf{h_R}$: $\mathbf{h_R} = \bar{h}_i$, where i =1, 2, 3 … n. the average holding cost per unit of item per unit time at retailer nodes.

$\mathbf{h_0}$: $\mathbf{h_D + h_R}$ the holding cost for integrated DC system.

$\mathbf{h_1}$: The holding cost per unit of item per unit time at WH.

$\mathbf{g_D}$: The unit shortage cost at DC.

$\mathbf{g_R}$: $\mathbf{g_R} = \bar{g}_i$, where i =1, 2, 3 … n. The average shortage cost per unit shortage at retailer nodes.

g : $\mathbf{g = g_D + g_R}$ The unit shortage cost for integrated DC system.

$\sum_{i=1}^{k} a_i = a_1 + a_2 + ... + a_k.$ $\quad \sum_{i=0}^{nQ}{}^{*}\, i = 0 + Q + 2Q + ... + nQ.$

3. Analysis

Let $I_0(t)$ and $I_1(t)$ denote the on-hand inventory levels at Distribution Centre and warehouse respectively at time t^+. From the assumptions on the input and output processes, $I(t) = \{(I_0(t), I_1(t) : t \geq 0\}$ is a Markov process with state space $E = \{(j, q) / j = S, (S-1), ..., s, (s-1), ..., 2, 1, 0., \text{ and } q = nQ, (n-1)Q, ..., Q\}$. Since E is finite and all its states are recurrent non-null, $\{I(t) : t \geq 0\}$ is an irreducible Markov process with state space E and it is an ergodic process. Hence the limiting distribution exists and is independent of the initial state.

The infinitesimal generator of this process $R = (a(j,q:k,r))_{(j,q),(k,r)\in E}$ can be obtained from the following arguments.

- The arrival of a demand (or perish) for an item at Distribution centre makes a state transition in the Markov process from (j, q) to (j-1, q) with intensity of transition $\lambda + j\gamma$ (where $\lambda = \lambda_i$, i = 1,2,3… n.)
- Replenishment of inventory at Distribution centre makes a state transition from (j, nQ) to (j + Q, (n-1)Q) with rate of transition μ (> 0).

The infinitesimal generator R is given by

$$R = \begin{pmatrix} A & B & 0 & \cdots & 0 & 0 \\ 0 & A & B & \cdots & 0 & 0 \\ 0 & 0 & A & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A & B \\ B & 0 & 0 & \cdots & 0 & A \end{pmatrix}$$

The entries of the block partition matrix R can be written as

$$[R]_{q\,x\,r} = \begin{cases} A & \text{if} \quad q = r, \qquad r = nQ, (n-1)Q, (n-2)Q, \ldots Q. \\ B & \text{if} \quad q = r + Q \qquad r = (n-1)Q, (n-2)Q \ldots 2Q. \\ B & \text{if} \quad q = nQ \\ 0 & \text{otherwise} \end{cases}$$

The sub matrices A and B are given by

$$[A]_{j\,x\,q} = \begin{cases} \lambda + j\gamma & \text{if} \;\; q = j-1, \quad j = S,\ S-1, S-2,\ \ldots 1 \\ -(\lambda + j\gamma) & \text{if} \;\; q = j \qquad j = S,\ S-1, S-2,\ \ldots\ (s+1) \\ -(\lambda + + j\gamma + \mu) & \text{if} \;\; q = j \qquad j = s, s-1, s-2, \ldots, 1 \\ -\mu & \text{if} \;\; q = j \qquad j = 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$[B]_{j\,x\,q} = \begin{cases} \mu & \text{if} \quad q = j + Q \quad j = s, s-1, s-2, \ldots, 0. \\ 0 & \text{otherwise} \end{cases}$$

3.1 (a) Transient analysis

Define the transition probability function $p_{j,q}(k, r\,; t) = \Pr\{\, (I_0(t), I_1(t) = (k, r) \,|\, (I_0(0), I_1(0)) = (j, q) \,\}$. The corresponding transient matrix function is given by $P(t) = (p_{j,q}(k, r\,; t))_{(j,q),(k,r)\in E}$ vector which satisfies the Kolmogorov-forward equation $P'(t) = P(t)R$, where R is the infinitesimal generator of the process $\{I(t), t \geq 0\}$. The above equation, together with initial condition $P(0) = I$, the solution can be expressed in the form $P(t) = P(0)e^{Rt} = e^{Rt}$, where the matrix expansion in power series form is $e^{Rt} = I + \sum_{n=1}^{\infty} \frac{R^n t^n}{n!}$.

3.1 (b) Steady state analysis:

The structure of the infinitesimal matrix R reveals that the state space E of the Markov process $\{I(t), t \geq 0\}$, is finite and irreducible. Let the limiting probability distribution of the inventory level process be

$\nu_j^q = \lim_{t\to\infty} pr\{(I_0(t), I_1(t) = (j, q)\}_{(j,q)\in E}$, where ν_j^q is the steady state probability that the system be in state (j, q).

Let $\nu = (\nu^{nQ}, \nu^{(n-1)Q}, \nu^{(n-2)Q}, \ldots, \nu^{Q}, \nu^{0})$ denote the steady state probability distribution where $\nu^q = \left(\nu_S^q, \nu_{S-1}^q, \ldots, \nu_s^q, \nu_{s-1}^q, \ldots, \nu_0^q.\right)$ for the system under consideration. For each (j, q), ν_j^q can be obtained by solving the matrix equation $\nu A = 0$ together with normalizing condition $\sum_{(j,q)\in E} \nu_j^q = 1$

Assuming ν_Q^Q = a, we obtain the steady state probabilities of the system states as

$^{iQ}\nu = (-1)^k\, a\, (BA)^k$, i = 1,2, … n ; k = n-i+1. where a = $e\left[\sum_{i=0}^{n-1} (-1)^i \, (BA^{-1})^i\right]^{-1}$.

4 Operating characteristics

In this section, we derive some important system performance measures for the system under steady state.

(a) Mean Reorder Rates:

The Mean reorder rate at Distribution centre (β_0) and Manufacturer (β_1) are given by

$$\beta_0 = (\lambda + (s+1)\gamma) \sum_{q=Q}^{nQ}{}^{*} \nu_{s+1}^{q} \quad \text{and} \quad \beta_1 = \mu \sum_{j=0}^{s} \nu_j^{Q} \tag{1}$$

(b) Mean Inventory Levels:

The mean inventory level in the steady state at Distribution centre (I_0) and Manufacturer (I_1) are given by

$$I_0 = \sum_{q=Q}^{nQ}{}^{*} \left(\sum_{j=0}^{S} j.\nu_j^{q} \right) \quad \text{and} \quad I_1 = \sum_{j=0}^{S} \left(\sum_{q=Q}^{nQ}{}^{*} q.\nu_j^{q} \right). \tag{2}$$

(c) Mean Shortage Rate:

The Shortages occurs only at retailer node. The shortage rate (α_0) is given by

$$\alpha_0 = \lambda \sum_{q=Q}^{nQ}{}^{*} \nu_0^{q} \, . \tag{3}$$

5. Algorithm

In this Section we design the algorithm to compute the long run expected inventory cost (for all the echelons).

Step -1: Determine the matrices A and B.

Step-2: Generate the R Matrix.

Step-3: Solve the system Π R = 0 with normalizing condition $\Sigma\Pi = 1$.

Step-4: Compute the operating characteristics.

Step-5: Compute the long run expected inventory cost.

6. Cost analysis

In this section we impose a cost structure for the proposed model and analyze it by the criteria of minimization of long run total expected cost per unit time.

The long run expected cost rate C(s, Q) is given by

$$C(s,Q) = h_0 I_0 + h_1 I_1 + k_0 \beta_0 + k_1 \beta_1 + g\alpha_0 \tag{4}$$

Substituting (1),(2) and (3) in (4) we get

$$C(s,Q) = h_0\left(\sum_{q=Q}^{nQ}{}^{*}\left(\sum_{j=0}^{S} j.\nu_j^q\right)\right) + h_1\left(\sum_{j=0}^{S}\left(\sum_{q=Q}^{nQ}{}^{*} q.\nu_j^q\right)\right) + k_0\left((\lambda + (s+1)\gamma)\sum_{q=Q}^{nQ}{}^{*}\nu_{s+1}^q\right)$$
$$+ k_1\left(\mu_0 \sum_{j=0}^{s} \nu_j^Q\right) + g\left(\lambda \sum_{q=Q}^{nQ}{}^{*}\nu_0^q\right)$$

.

8. **Concluding remarks**

In this paper we analyzed a continuous review inventory control system in a supply chain. The structure of the chain allows vertical movement of goods from distribution center to retailers. The model is deals with lost sales at DC and the supply from manufacturer is in terms of pockets and system handling a single perishable product. This model deals an integrated distribution system (IDS) consist with one DC and multiple retailers (n). In this model the demand at retailers node follows a independent Poisson distribution with rate λ_i (i = 1, 2, 3, , n). Scanners collect sales data at retailer nodes and Electronic Data Interchange (EDI) allows these data to be shared to DC. With the strong communication network and transport facility a unit of item is transferred to the corresponding retailer with negligible lead time. Future research may include positive lead time for the above IDS and also we study the same system with partial backlogging.

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