Analysis of inventory system with retrial and direct demands in supply chain

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Abstract

Continuous review inventory control of a single item at a single location had been considered by many researchers in the last two decades. We extend this inventory control strategy to two-echelon system, which is a building block for supply chain. A (s, S) type inventory system with Poisson demand and exponentially distributed lead times is assumed at retailer node. The items are supplied to the retailers and the direct customers (Direct demand), in packs of Q (= S-s) items from the distribution center which has instantaneous replenishment facility from an abundant source (manufacturer). The unsatisfied customers at retailer are entered into the orbit of finite capacity N. These orbiting demands retry for their demand after a random time which is assumed to be exponential distribution. The transient and steady state probability distribution and the operating characteristics are obtained explicitly. The measures of system performance in the steady state are obtained.

Keywords: *Continuous review inventory system, Supply Chain, Retrial Demand, (s. S) Policy, Optimization.*

Introduction

A supply chain may be defined as an integrated process wherein a number of various business entities (suppliers, distributors and retailers) work together in an effort to (1) acquire raw materials (2) process them and then produce valuable products and (3)transport these final product to retailers. The process and delivery of goods through this network needs efficient communication and transportation system. The supply chain is traditionally characterized by a forward flow of materials and products and backward flow of information. Over the last two decades, researchers and practitioners have primarily investigated the various process of supply chain individually.

A Complete review was provided by Benita M. Beamon (1998)[10]. Recently wever, there has been increasing attention placed on performance, design and analysis of the supply chain as a whole. HP's(Hawlett Packard) Strategic Planning and Modeling(SPaM) group initiated this kind of research in 1977. From practical stand point the supply chain concept arose from a number of changes in the manufacturing environment, including the rising costs of manufacturing, the shrinking resources of manufacturing bases, shortened product life cycles, the leveling of planning field within manufacturing, inventory driven costs (IDC) involved in distribution and the globalization of market economics.

With in manufacturing research, the supply chain concept grow largely out of two-stage multi-echelon inventory models, and it is important to note that considerable research in this area is based on the classic work of Clark and Scarf (1960)[1].

A complete review on this development were recorded by Federgruen (1993)[9]. Recent developments in two-echelon models may be found in Q. M. He, and E. M. Jewkes (2000) [11], Axsater.S (1993)[7] and Nahimas, S(1982)[4]. Continuous review Perishable inventory with instantaneous replenishment was considered by Kalpakam, S and Arivarignan, G(1988)[5]. A continuous review (s, S) policy with positive lead times in two echelon Supply Chain was considered by Krishnan.K and Elango.C. 2005 [17].

This paper deals with a simple supply chain that is modeled as a single warehouse and multiple retailer system handling a single product. In order to avoid the complexity, at the same time without loss of generality, we assumed identical demand pattern at each node. This restricts our study to design and analyze a tandem network of inventory, which is a building block for the whole supply chain system.

The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, both transient and steady state analysis are done. Section 4 deals with the operating characteristics of the system and section 5 deals with the cost analysis for the operation. The last section 7, concludes the paper.

2. The Model description

The inventory control system in supply chain considered in this paper is defined as follows. A product is supplied from warehouse to retailer who adopts (s, S)

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policy for maintaining inventory of his own. The demand at retailer node follows a Poisson process with parameter λ_r (> 0). The unsatisfied customers at retailer are entered into the orbit of finite capacity N. These orbiting demands retry for their demand after a random time with rate θ(>0). The Supply to the retailer in packets of Q = S-s items is administrated with exponential lead time having parameter μ (> 0). We assume that the supplier (ware house) acts as a distribution center which adopts (0,M) policy where M=nQ, Q=S-s, $n \in N$. The Direct demand at Distributor node follows a Poisson distribution with parameter λ_D(> 0). The replenishment of items in terms of pockets (from source) at distribution center is instantaneous.

In our model the maximum inventory levels M and S are fixed and reorder level s vary such that S-s = Q and M = nQ, n = 1,2,...[M/Q] where [x]denotes the integral part of the real number x. We fix the following notations for the forthcoming analysis part of our paper.

$[\boldsymbol{R}]_{ij}$: The element /sub matrix at (i,j)th position of $\boldsymbol{R}$.

0 : Zero matrix.

I : Identity matrix.

e : A column vector of 1's of appropriate dimension.

$$\sum_{i=1}^{k} a_i = a_1 + a_2 + \ldots + a_k .$$

$$\sum_{i=0}^{nQ}{}^{*}\, i = 0 + Q + 2Q + \ldots + nQ.$$

3. Analysis

Let $I_1(t)$ denote the on hand inventory level at retailer, $I_2(t)$ denote the number of customers in the orbit and $I_3(t)$ denote the on hand inventory level at Distribution center at time t^+. From the assumptions on the input and output process,

I(t) = {($I_1(t)$, $I_2(t)$), $I_3(t)$: t ≥ 0 }

is a Markov process with state space

E = {(i, j, k) / i = S, S-1 …3, 2, 1: j = 0,1,…,N : k = Q, 2Q…nQ}.

The infinitesimal generator of this process

$$A = (a(i, j, k : l, m, n))_{(i, j, k)(l.m.n)\in E}$$

can be obtained using the following arguments.

- The arrival of a demand makes a state transition in the Markov Process from (i, j, k) to (i-1, j, k) with intensity of transition λ_r(>0).
- The arrival of a retrial demand makes a state transition in the Markov Process from (i, j, k) to (i-1, j-1, k) with intensity of transition θ (>0).
- The arrival of a direct demand to DC makes a state transition in the Markov Process from (i, j, k) to (i, j, k-Q) with intensity of transition λ_D(>0).
- Replenishment of inventory at retailer node makes a transition from (i, j, k) to (i+Q, j, k-Q) with rate of transition μ (> 0).

The infinitesimal generator **R** is given by

$$\boldsymbol{R} = \begin{pmatrix} A & B & 0 & 0 & 0 & \ldots & 0 \\ 0 & A & B & 0 & 0 & \ldots & 0 \\ 0 & 0 & A & B & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ B & 0 & 0 & 0 & 0 & \ldots & A \end{pmatrix}$$

The entries of the matrix $[\boldsymbol{R}]_{pq}$ can be written as

$$[R] = \begin{cases} A & \text{if} \quad q = r, \; r = nQ, (n-1)Q, (n-2)Q, \ldots Q. \\ B & \text{if} \quad q = r + Q; \quad r = (n-1)Q, (n-2)Q \ldots 2Q. \\ B & \text{if} \quad q = r \; ; \quad r = nQ \\ 0 & \text{otherwise} \end{cases}$$

Then the sub matrices of A and B are given by

$$[A] = \begin{cases} A_1 \text{ if} & q = r \; ; \; r = N, \; N-1, \; N-2.,,,,1. \\ A_2 \text{ if} & q = r + Q \; ; \; r = N, N-1, \ldots..1 \\ A_3 \text{ if} & q = r \; ; \quad r = 0 \\ 0 & \text{otherwise} \end{cases}$$

And

$$[B] = \begin{cases} M & \text{if} \quad q = r \qquad r = N, N-1, N-2, \ldots 1, 0 \\ 0 & \text{otherwise} \end{cases}$$

3. 1 Transient analysis

Let $I_1(t)$ and $I_3(t)$ denote the on hand inventory level at retailer node and warehouse (distribution node) respectively, and $I_2(t)$ denote the number of customers in the orbit of size N, at time t . From the assumptions on the input and output process, we define

{I(t) : t ≥ 0} = {($I_1(t)$, $I_2(t)$), $I_3(t)$) : t ≥ 0 }

is a Markov process with state space

E = {(i, j, k) / i = S, S-1 …3, 2, 1: j = 0,1,…,N : k = Q, 2Q…nQ}.

Theorem 3.1.1

The vector process {I(t) : t ≥ 0} where I(t) = {($I_1(t)$, $I_2(t)$), $I_3(t)$) for t ≥ 0 is a continuous time Markov Chain with state space

E = {(i, j, k) / i = S, S-1 …3, 2, 1: j = 0,1,…,N : k = Q, 2Q…nQ}.

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Proof: The stochastic process $\{I(t) : t \geq 0\}$ has a discrete state space with order relation '$\leq$' that $(i, j, k) \leq (l, m, n)$ if and only if $i \leq l$, $j \leq m$ and $k \leq n$. To prove that $\{I(t) : t \geq 0\}$ is a Markov chain, first we do a transformation for state space E to E' such that $(i, j, k) \rightarrow i+j+k \in E'$, where $E' = \{Q, Q + 1, ..., Q + S, ..., nQ + S, nQ + S+1, nQ + S+2, …, nQ + S+N\}$.

Now we may realize that $\{I(t) : t \geq 0\}$ is a stochastic process with discrete state space E'.

The joint distribution of random variables $\{I(t_1), I(t_2), ..., I(t_n)\}$ and $\{I(t_1+\tau), I(t_2+\tau), ..., I(t_n+\tau)\}$ with $\tau > 0$ (an arbitrary real number) are equal.

In particular the conditional probability

$$\Pr\{I_n = k \mid I_{n-1} = j, I_{n-2} = i, ...I_0 = 1\} = \Pr\{I_n = k \mid I_{n-1} = j\}$$

due to the single step transition of states in E.

Hence $\{I(t) : t \geq 0\}$ is a continuous time Markov Chain.

Define the transition probability function

$$P_{i,j,k}(l, m, n : t) = \Pr\{(I_1(t), I_2(t), I_3(t)) = (l, m, n) \mid (I_1(0), I_2(0), I_3(0)) = (i, j, k)\}$$

The corresponding transition matrix function is given by

$$P(t) = (P_{i,j,k}(l, m, n : t))_{(i,j,k)(l,m,n)} \in E$$

which satisfies the Kolmogorov- forward equation

$$\mathbf{P'(t) = P(t)R}$$

where **R** is the infinitesimal generator.

From the above equation, together with initial condition P(0) = I, the solution can be expressed in the form

$$P(t) = P(0)e^{Rt} = e^{Rt}$$

where the matrix expansion in power series form is

$$\mathbf{e^{Rt} = I} + \sum_{n=1}^{\infty} \frac{R^n t^n}{n!}$$

case(i) : suppose that the Eigen values of R are all distinct. Then from the spectral theorem of matrices, we have

$$\mathbf{R = HDH^{-1}}$$

where H is the non-singular (formed with the right Eigen vectors of R) and D is the diagonal matrix having its diagonal elements the eigen values of R. Now 0 is an Eigen value of R and if $d_i \neq 0$, $i = 1, 2, ...,m$ are the distinct eigen values then

$$D = \begin{pmatrix} 0 & 0 & .. & .. & 0 \\ 0 & d_1 & .. & .. & ... \\ ... & ... & .. & .. & ... \\ 0 & 0 & .. & d_{m-1} & 0 \\ 0 & .. & .. & .. & d_m \end{pmatrix}$$

Then we have

$$D^n = \begin{pmatrix} 0 & 0 & .. & .. & 0 \\ 0 & (d_1)^n & .. & .. & ... \\ ... & ... & .. & .. & ... \\ 0 & 0 & .. & (d_{m-1})^n & 0 \\ 0 & .. & .. & .. & (d_m)^n \end{pmatrix}$$

and $R^n = HD^nH^{-1}$

Using R^n in P(t) we have the explicit solution of P(t) as

$$P(t) = He^{Dt}H^{-1} \quad \text{where}$$

$$e^{Dt} = \begin{pmatrix} 1 & 0 & .. & .. & 0 \\ 0 & e^{d_1 t} & .. & .. & ... \\ ... & ... & .. & .. & ... \\ 0 & 0 & .. & e^{d_{m-1}t} & 0 \\ 0 & .. & .. & .. & e^{d_m t} \end{pmatrix}$$

case(ii): Suppose the Eigen values of R are all not distinct, we can find a canonical representation as $R = SZS^{-1}$. From this the transition matrix P(t) can be obtained in a modified form. (Medhi, J[17]).

3. 2 Steady state analysis:

The structure of the infinitesimal matrix R reveals that the state space E of the Markov chain $\{I(t) : t \geq 0\}$, is finite and irreducible. Let the limiting distribution of the inventory level process be defined by

$$P_{i,j}^k = \lim_{t\to\infty} \Pr\{(I_1(t), I_2(t), I_3(t)) = (i, j, k)\}_{(i,j,k)\in E}$$

where $P_{i,j}^k$ is the steady state probability for the system be in state (i,j,k), (Cinlar [5]).

Let $\quad P = \left(P_{i,j}^{nQ}, P_{i,j}^{(n-1)Q}, P_{i,j}^{(n-2)Q} ... P_{i,j}^{Q}\right)$

denote the steady state probability distribution where I = 0, 1, 2, …. S: and J = 0 , 1, 2, …, N for the system under consideration. For each (i,j,k), $p^k_{i,j}$ can be obtained by solving the matrix equation $P.R = 0$ together with normalizing condition $\sum_{i,j,k} \mathbf{P_{i,j}^k} = \mathbf{1}$

Assuming $P_{i,j}^{Q} = a$, we Obtain the steady state probability

$$P^{iQ} = (-1)^k \, a \, (BA)^k, \; i = 1,2, \dots n \; ; \quad k = n-i+1.$$

$$\text{Where } a = e^1 \left[\sum_{i=0}^{n-1} (-1)^i \; (BA^{-1})^i\right]^{-1}$$

4. Operating characteristics

4.1 Reorder rate:

Consider the event β_i of reorders at retailer node and Distribution node .Observe that β_0 events occur whenever the inventory level at retailer node reaches s whereas the β_1 -event occurs whenever the inventory

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level at DC reaches 0. The mean reorder rate at retailer node is given by

$$\beta_0 = \lambda_r \sum_{q=Q}^{nQ} \sum_{j=1}^{N} P_{s+1,j}^{q}$$

The reorder rate at the DC is given by

$$\beta_1 = (\mu + \lambda_D) \sum_{i=0}^{s} \sum_{j=0}^{N} P_{i,j}^{q}$$

4.2 Mean inventory level:

Let I_0 denote the mean inventory level at retailer node and I_1 denote the mean inventory level at DC in the steady state. Thus, $\overline{I_0} = \sum_{q=Q}^{nQ} \sum_{j=0}^{N} \left(\sum_{i=0}^{S} i.p_{i,j}^{q} \right)$ and

$$\overline{I_1} = \sum_{i=0}^{N} \sum_{j=0}^{S} \left(\sum_{q=Q}^{nQ} q.p_{i,j}^{q} \right)$$

4.3 Shortage rate:

Shortages occur only at retailer node,. The shortage rate is given by

$$\alpha_0 = \lambda_r \sum_{q=Q}^{nQ} \sum_{j=0}^{N} P_{0,j}^{q}$$

4.4 Expected number of customer in the Orbit:

Let γ_0 denote the number of customers in the orbit which is given by

$$\gamma_0 = \sum_{q=Q}^{nQ} \sum_{i=0}^{S} \sum_{k=1}^{N} k.P_{i,j}^{q}$$

5 Cost analysis

In this section we analyze the cost structure for the proposed model by considering the minimization of the steady state total expected cost per unit time. There are fixed ordering costs k_1 associated with each order initiated from warehouse to source and k_0 that of initiated from retailer to warehouse (distribution) regardless of the order size.

The holding cost h_1 per unit of item per unit time at warehouse and the holding cost h_0 per unit of item per unit time at retailer node are considered. The demands occurs during stock out period are assumed to be lost and **g** be the shortage cost per unit shortage at retailer node. The long run expected cost rate C(s,Q) is given by

$$C(s,Q) = h_0 \overline{I_0} + h_1 \overline{I_1} + k_0\beta_0 + k_1\beta_1 + \alpha_0 g + \gamma_0 b$$

where h_i is the holding cost at node i , k_i is the setup cost for node i (i=0,1) ,g is the shortage cost for unit shortage and b is the backordering cost of a demand in the orbit per unit time. Although we have not proved analytically the convexity of the cost function C(s,Q), our experience with considerable number of numerical examples indicates that C(s,Q) for fixed Q to be convex in s. In some cases it turned out to be an increasing function of s. Hence we adopted the numerical search procedure to determine the optimal values s^* and b^*, consequently we obtain optimal $n^* = \left[\frac{M}{Q^*} \right]$.For large number of parameters, our calculation of C(s, Q) revealed a convex structure for the same.

6. Concluding remarks

In this paper we analyzed a continuous review inventory control system with retrial and direct demand in a supply chain. The structure of the chain allows vertical movement of goods from distribution center to retailers. The model is dealing lost sales at retailer node and the supply from distribution center is in terms of pockets. This model deals with only tandem network (basic structure of supply chain), this structure can be extended to tree structure and to more general.

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